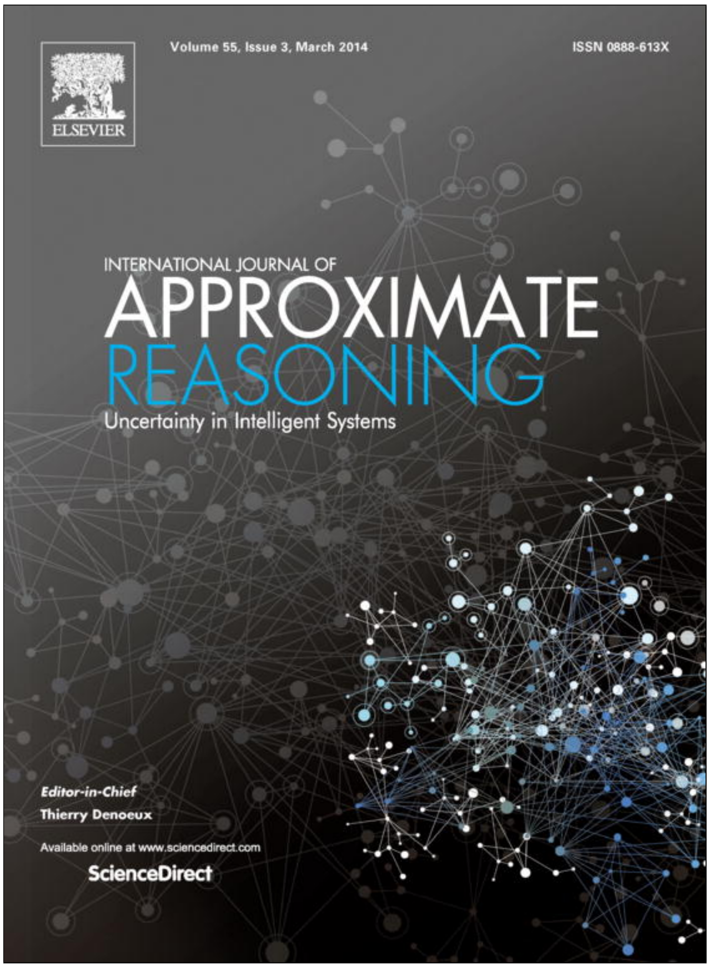
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A novel cognitive system model and approach to transformation of information granules

Weihua Xu *, Jinzhong Pang, Shuqun Luo

School of Mathematics and Statistics, Chongqing University of Technology, Chongqing, 400054, PR China

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ABSTRACT

In this paper, a novel cognitive system model is established based on formal concept analysis to exactly describe human cognitive processes. Two new operators, extent–intent and intent–extent, are introduced between an object and its attributes. By analyzing the necessity and sufficient relations between the object and some of its attributes, the information granule concept is investigated in human cognitive processes. Furthermore, theories of transforming arbitrary information granule into necessary, sufficient, sufficient and necessary information granules are addressed carefully. Algorithm of the transformation is constructed, by which we can provide an efficient approach to the conversion among information granules. To interpret and help understand the theories and algorithm, an experimental computing program is designed and two cases is employed as case study. Results of the small scale case are calculated by the method presented in this paper. The large-scale case is calculated by the experimental computing program and validated by the proposed algorithm. The considered framework can provide a novel convenient tool for artificial intelligence researches.

1. Introduction

Currently, every type of system is becoming increasingly complex with the rapid growth of automated production. Traditional manual labor has been replaced with high-performance computers. Human cognition is becoming the bottleneck of further productive force development. Artificial intelligence has been a very hot topic regarding how to improve the efficiency and quality of human thinking. Generally, there exist two research methods in artificial intelligence, "top-down" and "bottom-up" [25]. Although artificial intelligence has made great achievements in logic simulation and each method possesses its own unique advantages, such simulation methods represent very basic and low-level cognitive systems. Thus, a system that combines the strength of the "top-down" and "bottom-up" strategies is required.

Recently, there have been many advances in the study for formal concept analysis and rough set with granular computing. Ma and Zhang [18] presented a general framework for concept lattice and established generalized concept systems based on set-theoretic operators are established. Li et al. [13] studied the issues of approximate concept construction, rule acquisition and knowledge reduction in incomplete decision contexts. What's more, Shen and Zhang [15] explored the relationship between contexts, closure spaces, and complete lattices. Scott Dick et al. [5] established a novel architecture for a granular neural network and Zhang and Miao [43] investigated two basic double-quantitative rough set models of precision and grade and their investigation using granular computing. What's more, Andrzej Bargiela [1,2] researched the roots of granular computing and expanded the theory of granular computing for human-centered information processing. And Yao [33–37]

* Corresponding author.
E-mail address: chxuwh@gmail.com (W.H. Xu).

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discussed integrative levels of granularity and the theory of granular computing. Many excellent achievements can be seen in Ref. [42].

Information granules have played a significant role in human cognitive processes. Information granules refer to pieces, classes, and groups into which complex information is divided in accordance with the characteristics and processes of human understanding and decision-making. Such a process is termed "information granulation." Zadeh first proposed and discussed the issue of fuzzy information granulation [38] in 1979. Since then, the basic idea of information granulation [19] has been applied to many fields, such as the theory of rough sets [21,22], fuzzy sets [39], and evidence theories [24]. In 1985, Hobbs proposed the concept of granularity [10], and Zadeh first explored the concept of granular computing [38] between 1996 and 1997. Currently, granular computing serves an important role in soft computing, knowledge discovery, and data mining, providing excellent results [12,14].

German mathematician R. Wille proposed the theory of formal concept analysis (FCA) [28] in 1982. This theory is based on mathematical expressions of the formal concept and is a branch of applied mathematics [6]. The application of mathematical methods to manage conceptual data and knowledge using formal concept analysis theory is necessary. A concept lattice is an ordered hierarchy that is defined by a binary relationship between the objects and attributes in a data set. Concepts are a reflection of cognitive processes, which are driven by concepts. Most of the research on the concept lattice concentrates on topics, such as concept lattice construction [3,4,7,9,11,17], concept lattice pruning [20], rule acquisition, the relationship between the concept lattice and rough set [23,32], and applications [16,26,30,31]. Formal concept analysis is becoming a powerful tool for data analyses and knowledge processing and has successfully been applied to many fields [8,27,40,41], such as knowledge engineering, information searching, and software engineering.

From granular computing theories and formal concept analysis theories, the cognitive process is a process of transformation between the object and its easily identifiable attributes. Humans begin to recognize objects from the unknown. If an object is of interest, it will possess a fuzzy and rough impression on initial perception. The uncertainty impression is composed from some of the major sufficient or necessary attributes. The object can then be further understood after learning its attributes, and the attributes can be further judged after understanding the object. Finally, the sufficient and necessary attributes of the object can be obtained, grasping the object completely.

To describe human cognitive processes, this paper will establish a novel model of a cognitive system based on formal concept analysis and discuss information granules in the cognitive system.

This paper is organized as follows. Some preliminary concepts of FCA that are required in our work are briefly reviewed in Section 2. In Section 3, we propose two operators between an object and its attributes, extent–intent and intent–extent operators, and a novel model of a cognitive system is then constructed based on FCA. In Section 4, concepts of information granules, such as necessary information, sufficient information, and necessary and sufficient information granules, are presented. Moreover, the relationship between an object and its attributes is discussed in view of information granules in this cognitive system. In the next section, we suggest how to transform arbitrary information granule into necessary information granules, sufficient information granules and sufficient and necessary information granules. Furthermore, we design the program to transform arbitrary information granule and investigate an interesting case analysis about how to make decisions in Section 6. Finally, we conclude our contribution with a summary and an outlook for further research.

2. Formal concept analysis

Generally, the data analyzed by the concept lattice are represented in a formal context, which is defined as follows.

Definition 1. (See [6].) A triple (U, A, I) is a formal context if U and A are sets and $I \subseteq U \times A$ is a binary relation between U and A, where $U = \{x_1, x_2, \ldots, x_n\}$ and $A = \{a_1, a_2, \ldots, a_m\}$ are called the object and attribute sets, respectively. Each x_i $(i \leqslant n)$ and a_j $(j \leqslant m)$ is an object and attribute, respectively.

In a formal context (U, A, I), if $(x, a) \in I$, also written as xIa, then the object x has the attribute a, or a is possessed by the object x. In this paper, $(x, a) \in I$ is denoted by 1 and $(x, a) \notin I$ is denoted by 0. Thus, the formal context can be represented using a table with only 0 and 1.

With respect to the formal context (U, A, I), a pair of dual operators can be defined as follows. For $X \subseteq U$ and $B \subseteq A$,

(1) $X^* = \{a \mid a \in A,\ \forall x \in X,\ xIa\}$,
(2) $B^* = \{x \mid x \in U,\ \forall a \in B,\ xIa\}$,

where X^* is the set of attributes shared by all objects in X and B^* is the set of objects that possesses all attributes in B. We write $\{x\}^*$ as x^* for any $x \in U$ and $\{a\}^*$ as a^* for any $a \in A$. If for any $x \in U$, $x^* \neq \emptyset$ and $x^* \neq A$, and for any $a \in A$, $a^* \neq \emptyset$ and $a^* \neq U$, then the formal context (U, A, I) is canonical. Without this generalization, the formal contexts are all canonical contexts in this paper.

Definition 2. (See [6].) Let (U, A, I) be a formal context. A pair (X, B) is called a formal concept, or concept, if and only if $X^* = B$ and $B^* = X$, where X is the extension and B is the intension of (X, B).

Example 1. Table 1 shows a formal context (U, A, I) in which $U = \{x_1, x_2, x_3, x_4\}$ and $A = \{a, b, c, d\}$.

Table 1
A formal context (U, A, I).

U	a	b	c	d
x_1	1	0	1	1
x_2	1	1	0	0
x_3	0	0	1	0
x_4	1	1	0	0

From this context, we can write the following concepts:

$$(U, \emptyset),$$
$$\big(\{x_1, x_2, x_4\}, \{a\}\big),$$
$$\big(\{x_2, x_4\}, \{a, b\}\big),$$
$$\big(\{x_1\}, \{a, c, d\}\big),$$
$$\big(\{x_1, x_3\}, \{c\}\big),$$
$$(\emptyset, A).$$

For a formal context (U, A, I), the following significant properties hold.

Proposition 1. *(See [6].) Let (U, A, I) be a formal context, for any $X_1, X_2, X \subseteq U$, $B_1, B_2, B \subseteq A$, the seven properties hold as follows:*

(1) $X_1 \subseteq X_2 \Rightarrow X_2^* \subseteq X_1^*$, $B_1 \subseteq B_2 \Rightarrow B_2^* \subseteq B_1^*$;
(2) $X \subseteq X^{**}$, $B \subseteq B^{**}$;
(3) $X^* = X^{***}$, $B^* = B^{***}$;
(4) $X \subseteq B^* \Leftrightarrow B \subseteq X^*$;
(5) $(X_1 \cup X_2)^* = X_1^* \cap X_2^*$, $(B_1 \cup B_2)^* = B_1^* \cap B_2^*$;
(6) $(X_1 \cap X_2)^* \supseteq X_1^* \cup X_2^*$, $(B_1 \cap B_2)^* \supseteq B_1^* \cup B_2^*$;
(7) (X^{**}, X^*) *and* (B^*, B^{**}) *are concepts.*

3. Cognitive system based on formal concept analysis

Intrinsically, a cognitive process is the transformation process between an object and its attributes. In other words, men judge and recognize any object by using its transformation. When an object is consistent with its attributes, the laws of the object can be grasped. Thus, some of the necessary or sufficient information about the object can be gradually obtained. The object is constantly judged using this information until an understanding of the necessary and sufficient attributes of the object is gained.

In the next section, we will propose the operators between an object and its attributes to construct cognitive systems based on formal concepts.

Let L be a lattice, where 0_L and 1_L are zero and the unit element, respectively.

Definition 3. Let L_1 and L_2 be two complete lattices, for any $a_1, a_2 \in L_1$. $\mathcal{L} : L_1 \to L_2$ is an extent–intent operator if $\mathcal{L}$ satisfies the following:

(1) $\mathcal{L}(0_{L_1}) = 1_{L_2}$, $\mathcal{L}(1_{L_1}) = 0_{L_2}$,
(2) $\mathcal{L}(a_1 \vee a_2) = \mathcal{L}(a_1) \wedge \mathcal{L}(a_2)$.

For any $a \in L_1$, we say that $\mathcal{L}(a)$ is an intent element of a and elements of L_2 are intent elements.

Moreover, for any $b_1, b_2 \in L_2$, $\mathcal{H} : L_2 \to L_1$ is an intent–extent operator if $\mathcal{H}$ satisfies the following:

(1′) $\mathcal{H}(0_{L_2}) = 1_{L_1}$, $\mathcal{H}(1_{L_2}) = 0_{L_1}$,
(2′) $\mathcal{H}(b_1 \vee b_2) = \mathcal{H}(b_1) \wedge \mathcal{H}(b_2)$.

For any $b \in L_2$, we say that $\mathcal{H}(b)$ is extent element of b and elements of L_1 are extent elements.

Definition 4. A quadruplet $(L_1, L_2, \mathcal{L}, \mathcal{H})$ is a cognitive system if the above two operators, $\mathcal{L}$ and $\mathcal{H}$, further satisfy

$$\mathcal{H} \circ \mathcal{L}(a) \geqslant a$$

and

$$\mathcal{L} \circ \mathcal{H}(b) \geqslant b,$$

where $\mathcal{H} \circ \mathcal{L}(a)$ and $\mathcal{L} \circ \mathcal{H}(b)$ represent $\mathcal{HL}(a)$ and $\mathcal{LH}(b)$, respectively.

From the above definition, we can find that two operators, $\mathcal{L}$ and $\mathcal{H}$, characterize an object and its attributes in a cognitive process.

Theorem 1. *For any $a_1, a_2 \in L_1$ and $b_1, b_2 \in L_2$, a cognitive system $(L_1, L_2, \mathcal{L}, \mathcal{H})$ has the following properties*:

(1) *If $a_1 \leqslant a_2$, then $\mathcal{L}(a_2) \leqslant \mathcal{L}(a_1)$*;
(2) *If $b_1 \leqslant b_2$, then $\mathcal{H}(b_2) \leqslant \mathcal{H}(b_1)$*;
(3) $\mathcal{L}(a_1) \vee \mathcal{L}(a_2) \leqslant \mathcal{L}(a_1 \wedge a_2)$;
(4) $\mathcal{H}(b_1) \vee \mathcal{H}(b_2) \leqslant \mathcal{H}(b_1 \wedge b_2)$;
(5) $b \leqslant \mathcal{L}(a) \Leftrightarrow a \leqslant \mathcal{H}(b)$, $\mathcal{L}(a) \leqslant b \Leftrightarrow \mathcal{H}(b) \leqslant a$;
(6) *For any $a \in L_1$, $\mathcal{L} \circ \mathcal{H} \circ \mathcal{L}(a) = \mathcal{L}(a)$*;
(7) *For any $b \in L_2$, $\mathcal{H} \circ \mathcal{L} \circ \mathcal{H}(b) = \mathcal{H}(b)$.*

Proof. (1), (2), (3), (4) and (5) can be proven by Definition 3 directly.

(6) By (1) and $\mathcal{H} \circ \mathcal{L}(a) \geqslant a$, we can obtain $\mathcal{L}(a) \geqslant \mathcal{L} \circ \mathcal{H} \circ \mathcal{L}(a)$. In contrast, from $\mathcal{L} \circ \mathcal{H}(b) \geqslant b$, if we take $b = \mathcal{L}(a)$, then we can obtain $\mathcal{L} \circ \mathcal{H} \circ \mathcal{L}(a) \geqslant \mathcal{L}(a)$.

Thus, $\mathcal{L} \circ \mathcal{H} \circ \mathcal{L}(a) = \mathcal{L}(a)$.

(7) It can be proven in a manner similar to (6).

The proof is completed. □

From the above proof and FCA, we can obtain the following important results.

Theorem 2. *Let (U, A, I) be a formal context, where $U = \{x_1, x_2, \ldots, x_n\}$ and $A = \{a_1, a_2, \ldots, a_m\}$. If $L_1 = P(U)$ and $L_2 = P(A)$ are denoted, then the operators $(*, *)$ defined by Definition 1(1), (2) are extent–intent and intent–extent operators of (U, A, I), respectively.*

Proof. Theorem 2 can be obtained using Proposition 1 and Definition 3. □

From the above discussion, the formal context can be viewed as the relationship between the objects and attributes in a cognitive process. Some concepts that have been identified have been realized. Concepts that have not yet been recognized are not formed. To gain more knowledge about the cognitive process, we must find two extension and intension operators in accordance with Definition 3. In fact, these operators are "$*$" in a formal context.

4. Information granules in cognitive systems

In the previous section, we established two intension and extension operators between the objects and attributes. When using these two operators, the transformation process of an object and its attributes in the cognitive process can be understood. When the object is consistent with its attributes, we can grasp the nature or the laws of the object. Humans begin to understand things from the unknown. Thus, sufficient or necessary attributes of the unknown objects can be obtained using the two operators.

In this section, we will discuss the relationship between the object and their attributes in the cognitive process based on information granules using the two operators.

To reflect on the granule description of the cognitive system, the pair (a, b) denotes the information granule, where a is an object set and b is an attribute set.

Definition 5. Let $L_1 = P(U)$ and $L_2 = P(A)$ be two complete lattices and $\mathcal{L}$, $\mathcal{H}$ be extent–intent and intent–extent operators, respectively (i.e., $(L_1, L_2, \mathcal{L}, \mathcal{H})$ is a cognitive system). For any $a \in L_1$ and $b \in L_2$, denote

$$\mathcal{G}_1 = \big\{(a, b) \bigm| b \leqslant \mathcal{L}(a),\ a \leqslant \mathcal{H}(b)\big\}, \qquad \mathcal{G}_2 = \big\{(a, b) \bigm| \mathcal{L}(a) \leqslant b,\ \mathcal{H}(b) \leqslant a\big\}.$$

If $(a, b) \in \mathcal{G}_1$, then (a, b) is a necessary information granule of the cognitive system and b is a necessary attribute of object a (Fig. 1). Simultaneously, $\mathcal{G}_1$ is a necessary information granule set of the cognitive system.

If $(a, b) \in \mathcal{G}_2$, then (a, b) is a sufficient information granule of the cognitive system and b is a sufficient attribute of object a (Fig. 2). Simultaneously, $\mathcal{G}_2$ is a sufficient information granule set of the cognitive system.

If $(a, b) \in \mathcal{G}_1 \cap \mathcal{G}_2$, that is, (a, b), satisfy $b = \mathcal{L}(a)$ and $a = \mathcal{H}(b)$, then (a, b) is a sufficient and necessary information granule of the cognitive system and b is a sufficient and necessary attribute of a.

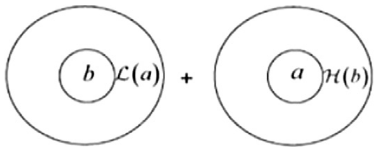

Fig. 1. (a, b) is a necessary information granule, and b is a necessary attribute of object a.

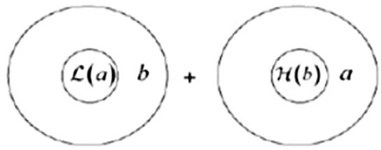

Fig. 2. (a, b) is a sufficient information granule, and b is a sufficient attribute of object a.

If $(a, b) \in \mathcal{G}_1 \cup \mathcal{G}_2$, then (a, b) is an information granule of the cognitive system and $\mathcal{G}_1 \cup \mathcal{G}_2$ is an information granule set of the cognitive system.

If $(a, b) \notin \mathcal{G}_1 \cup \mathcal{G}_2$, then (a, b) is an inconsistent information granule of the cognitive system, in which the facts shown in Figs. 1 and 2 are not generally true.

From the above definition, the necessary and sufficient information granules are concepts of the cognitive system. In fact, these concepts are also targets in human cognitive processes. In the real world, we first learn the necessary or sufficient information granules. We then gradually seek for the necessary and sufficient information granules, namely, the concept, based on the already known information granules.

Theorem 3. *Let (U, A, I) be a formal context, where $U = \{x_1, x_2, \ldots, x_n\}$ and $A = \{a_1, a_2, \ldots, a_m\}$. If $L_1 = P(U)$ and $L_2 = P(A)$ are denoted, the following hold for any $X \in L_1$ and $B \in L_2$:*

(1) *If $X^* \supseteq B$ and $B^* \supseteq X$, then B is a necessary attribute of X;*
(2) *If $X^* \subseteq B$ and $B^* \subseteq X$, then B is a sufficient attribute of X.*

Proof. Theorem 3 can be achieved directly using Definition 5 and Theorem 2. $\square$

From the above discussion, Definition 5 and Theorem 3 have shown that we seek to understand sufficient or necessary attributes when the concept may not be precise in cognitive processes.

Theorem 4. *Let $(L_1, L_2, \mathcal{L}, \mathcal{H})$ be a cognitive system and $\mathcal{G}_1$ be a necessary information granule set of the cognitive system. If "$\wedge$" and "$\vee$" are defined operators of $\mathcal{G}_1$, and*

$$(a_1, b_1) \wedge (a_2, b_2) = \big(a_1 \wedge a_2, \mathcal{L} \circ \mathcal{H}(b_1 \vee b_2)\big),$$
$$(a_1, b_1) \vee (a_2, b_2) = \big(\mathcal{H} \circ \mathcal{L}(a_1 \vee a_2), b_1 \wedge b_2\big),$$

$(\mathcal{G}_1, \leqslant)$ is then closed with respect to operators "$\wedge$" and "$\vee$".

Proof. Assume $(a_1, b_1), (a_2, b_2) \in \mathcal{G}_1$, then

$$b_1 \leqslant \mathcal{L}(a_1), \qquad b_2 \leqslant \mathcal{L}(a_2),$$
$$a_1 \leqslant \mathcal{H}(b_1), \qquad a_2 \leqslant \mathcal{H}(b_2)$$

and

$$a_1 \wedge a_2 \leqslant \mathcal{H}(b_1) \wedge \mathcal{H}(b_2) = \mathcal{H}(b_1 \vee b_2) = \mathcal{H} \circ \mathcal{L} \circ \mathcal{H}(b_1 \vee b_2).$$

Moreover, using Theorem 1 we find

$$\mathcal{L} \circ \mathcal{H}(b_1 \vee b_2) = \mathcal{L}\big(\mathcal{H}(b_1) \wedge \mathcal{H}(b_2)\big) \leqslant \mathcal{L}(a_1 \wedge a_2).$$

Thus, $(a_1, b_1) \wedge (a_2, b_2)$ is a necessary information granule; that is, $(a_1, b_1) \wedge (a_2, b_2) \in \mathcal{G}_1$.

$(a_1, b_1) \vee (a_2, b_2) \in \mathcal{G}_1$ can be proven similarly.

The theorem is proven. $\square$

Theorem 5. *Let $(L_1, L_2, \mathcal{L}, \mathcal{H})$ be a cognitive system and $\mathcal{G}_2$ be a sufficient information granule set of the cognitive system. If "$\wedge$" and "$\vee$" are defined operators of $\mathcal{G}_2$, and*

$$(a_1, b_1) \wedge (a_2, b_2) = \big(a_1 \wedge a_2, \mathcal{L} \circ \mathcal{H}(b_1 \vee b_2)\big),$$
$$(a_1, b_1) \vee (a_2, b_2) = \big(\mathcal{H} \circ \mathcal{L}(a_1 \vee a_2), b_1 \wedge b_2\big),$$

$(\mathcal{G}_2, \leqslant)$ is then closed with respect to operators "$\wedge$" and "$\vee$".

Proof. Let $(a_1, b_1), (a_2, b_2) \in \mathcal{G}_2$, then

$$\mathcal{L}(a_1) \leqslant b_1, \qquad \mathcal{L}(a_2) \leqslant b_2,$$
$$\mathcal{H}(b_1) \leqslant a_1, \qquad \mathcal{H}(b_2) \leqslant a_2$$

and

$$\mathcal{H} \circ \mathcal{L} \circ \mathcal{H}(b_1 \vee b_2) = \mathcal{H}(b_1 \vee b_2) = \mathcal{H}(b_1) \wedge \mathcal{H}(b_2) \leqslant a_1 \wedge a_2.$$

Moreover, using Theorem 1, we find

$$\mathcal{L}(a_1 \wedge a_2) \leqslant \mathcal{L}\big(\mathcal{H}(b_1) \wedge \mathcal{H}(b_2)\big) = \mathcal{L} \circ \mathcal{H}(b_1 \vee b_2).$$

Thus, $(a_1, b_1) \wedge (a_2, b_2)$ is a sufficient information granule; that is, $(a_1, b_1) \wedge (a_2, b_2) \in \mathcal{G}_2$.

$(a_1, b_1) \vee (a_2, b_2) \in \mathcal{G}_2$ can be proven similarly.

The theorem is proven. □

Because $\leqslant$ is a quasi-order relationship in $(\mathcal{G}_1, \leqslant)$ and $(\mathcal{G}_2, \leqslant)$, these relationships are not lattices with respect to operators "$\wedge$" and "$\vee$", termed "quasi-lattices".

5. Transformation of information granules

As previously described, we began to learn from the unknown. In other words, sufficient and necessary information granules do not exist at the beginning of cognitive systems. We will present approaches to transform useless information into very useful information granules in the following. That's to say, we can transform general information granules into necessary information granules, sufficient information granules, sufficient and necessary information granules.

Case 1. The method to transform the general information granules into necessary information granules can be represented in the following.

Theorem 6. *Let $(L_1, L_2, \mathcal{L}, \mathcal{H})$ be a cognitive system and $\mathcal{G}_1$ be a necessary information granule set. If $a \in L_1$, $b \in L_2$, then*

(1) $(a \wedge \mathcal{H}(b), b \vee \mathcal{L}(a)) \in \mathcal{G}_1$;
(2) $(a \vee \mathcal{H}(b), b \wedge \mathcal{L}(a)) \in \mathcal{G}_1$;
(3) $(\mathcal{H}(b), b \wedge \mathcal{L}(a)) \in \mathcal{G}_1$;
(4) $(a \wedge \mathcal{H}(b), \mathcal{L}(a)) \in \mathcal{G}_1$;
(5) $(\mathcal{H} \circ \mathcal{L}(a), b \wedge \mathcal{L}(a)) \in \mathcal{G}_1$;
(6) $(a \wedge \mathcal{H}(b), \mathcal{L} \circ \mathcal{H}(b)) \in \mathcal{G}_1$.

Proof. (1) Because $(L_1, L_2, \mathcal{L}, \mathcal{H})$ is a cognitive system, from Theorem 1 and Definition 5 we have

$$\mathcal{L}\big(a \wedge \mathcal{H}(b)\big) \geqslant \mathcal{L}(a) \vee \mathcal{L}\big(\mathcal{H}(b)\big) \geqslant \mathcal{L}(a) \vee b$$

and

$$\mathcal{H}\big(b \vee \mathcal{L}(a)\big) = \mathcal{H}(b) \wedge \mathcal{H}\big(\mathcal{L}(a)\big) \geqslant a \wedge \mathcal{H}(b).$$

Thus, $(a \wedge \mathcal{H}(b), b \vee \mathcal{L}(a)) \in \mathcal{G}_1$.

Furthermore, we can obtain $(a \wedge \mathcal{H}(b), b \vee \mathcal{L}(a)) \in \mathcal{G}_1$.

(2) It can be proven in a manner similar to (1).

(3) Because $(L_1, L_2, \mathcal{L}, \mathcal{H})$ is a cognitive system, from Theorem 1 and Definition 5 we have

$$\mathcal{L} \circ \mathcal{H}(b) \geqslant b \geqslant \mathcal{L}(a) \wedge b$$

and

$$\mathcal{H}\big(b \wedge \mathcal{L}(a)\big) \geqslant \mathcal{H} \circ \mathcal{L} \circ \mathcal{H}(b) = \mathcal{H}(b).$$

Thus, $(\mathcal{H}(b), b \wedge \mathcal{L}(a)) \in \mathcal{G}_1$.

Moreover, we have $(\mathcal{H}(b), b \wedge \mathcal{L}(a)) \in \mathcal{G}_1$.

(4) It can be proven in a manner similar to (3).

(5) Because $(L_1, L_2, \mathcal{L}, \mathcal{H})$ is a cognitive system, from Theorem 1 and Definition 5 we have $\mathcal{L} \circ \mathcal{H} \circ \mathcal{L}(a) = \mathcal{L}(a) \geqslant \mathcal{L}(a) \wedge b$; and $\mathcal{H}(\mathcal{L}(a) \wedge b) \geqslant \mathcal{H} \circ \mathcal{L}(a) \vee \mathcal{H}(b) \geqslant \mathcal{H} \circ \mathcal{L}(a)$. Thus, $(\mathcal{H} \circ \mathcal{L}(a), b \wedge \mathcal{L}(a)) \in \mathcal{G}_1$.

Therefore, $(\mathcal{H} \circ \mathcal{L}(a), b \wedge \mathcal{L}(a)) \in \mathcal{G}_1$.

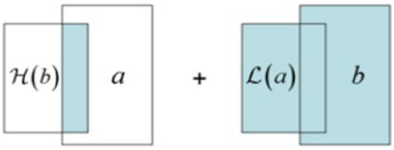

Fig. 3. $(a \wedge \mathcal{H}(b), b \vee \mathcal{L}(a))$ is a necessary information granule.

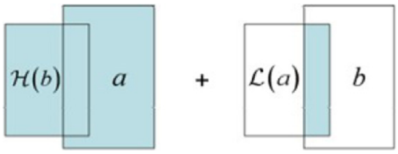

Fig. 4. $(a \vee \mathcal{H}(b), b \wedge \mathcal{L}(a))$ is a necessary information granule.

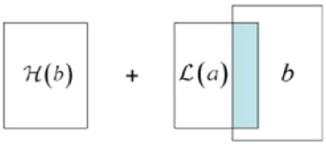

Fig. 5. $(\mathcal{H}(b), b \wedge \mathcal{L}(a))$ is a necessary information granule.

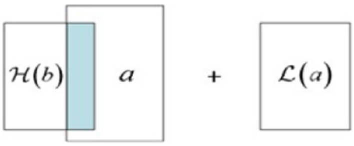

Fig. 6. $(a \wedge \mathcal{H}(b), \mathcal{L}(a))$ is a necessary information granule.

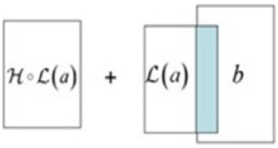

Fig. 7. $(\mathcal{H} \circ \mathcal{L}(a), b \wedge \mathcal{L}(a))$ is a necessary information granule.

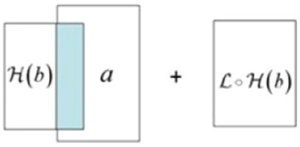

Fig. 8. $(a \wedge \mathcal{H}(b), \mathcal{L} \circ \mathcal{H}(b))$ is a necessary information granule.

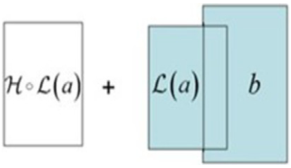

Fig. 9. $(\mathcal{H} \circ \mathcal{L}(a), b \vee \mathcal{L}(a))$ is a sufficient information granule.

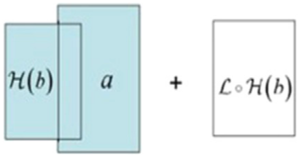

Fig. 10. $(a \vee \mathcal{H}(b), \mathcal{L} \circ b\mathcal{H}(b))$ is a sufficient information granule.

(6) It can be proven in a manner similar to (5). □

This theorem can be understood easily by using Figs. 3, 4, 5, 6, 7, 8.

Case 2. The method to transform the general information granules into sufficient information granules can be represented in the following.

Theorem 7. *Let $(L_1, L_2, \mathcal{L}, \mathcal{H})$ be a cognitive system and $\mathcal{G}_2$ be a sufficient information granule set. If $a \in L_1$, $b \in L_2$, then*

(1) $(\mathcal{H} \circ \mathcal{L}(a), b \vee \mathcal{L}(a)) \in \mathcal{G}_2$;
(2) $(a \vee \mathcal{H}(b), \mathcal{L} \circ \mathcal{H}(b)) \in \mathcal{G}_2$.

Proof. (1) Because $(L_1, L_2, \mathcal{L}, \mathcal{H})$ is a cognitive system, from Theorem 1 and Definition 5 we have

$$\mathcal{L} \circ \mathcal{H} \circ \mathcal{L}(a) = \mathcal{L}(a) \leqslant \mathcal{L}(a) \vee b,$$

and

$$\mathcal{H}\big(\mathcal{L}(a) \wedge b\big) = \mathcal{H} \circ \mathcal{L}(a) \wedge \mathcal{H}(b) \leqslant \mathcal{H} \circ \mathcal{L}(a).$$

Thus, $(\mathcal{H} \circ \mathcal{L}(a), b \vee \mathcal{L}(a)) \in \mathcal{G}_2$.

Furthermore, we achieve $(\mathcal{H} \circ \mathcal{L}(a), b \vee \mathcal{L}(a)) \in \mathcal{G}_2$.

(2) It can be proven in a manner similar to (1). □

This theorem can be illustrated by Figs. 9, 10.

From the above, we can transform useless information into very useful information granules in the cognitive system. If we do not receive the necessary and sufficient information granules in the cognitive system, we cannot fully recognize information granules given.

To fully learn information granules, we will show how to transform necessary, sufficient information granules into sufficient and necessary information granules respectively.

Case 3. The method to transform the necessary information granules into sufficient and necessary information granules can be represented in the following.

Theorem 8. *Let $(L_1, L_2, \mathcal{L}, \mathcal{H})$ be a cognitive system and $\mathcal{G}_1$ be a necessary information granule set. If $(a_1, b_1) \in \mathcal{G}_1$,*

(1) $(a_1 \vee \mathcal{H}(b_1), \mathcal{L}(a_1 \vee \mathcal{H}(b_1))) \in \mathcal{G}_1 \cap \mathcal{G}_2$;
(2) $(\mathcal{H}(b_1 \vee \mathcal{L}(a_1)), b_1 \vee \mathcal{L}(a_1)) \in \mathcal{G}_1 \cap \mathcal{G}_2$.

Proof. (1) By $(a_1, b_1) \in \mathcal{G}_1$, we have $a_1 \leqslant \mathcal{H}(b_1)$ and $b_1 \leqslant \mathcal{L}(a_1)$.
Thus,

$$a_1 \vee \mathcal{H}(b_1) = \mathcal{H}(b_1), \qquad \mathcal{L}\big(a_1 \vee \mathcal{H}(b_1)\big) = \mathcal{L} \circ \mathcal{H}(b_1).$$

Thus, from the above discussion and Theorem 1, we have the following:

$$\mathcal{L}\big(a_1 \vee \mathcal{H}(b_1)\big) = \mathcal{L} \circ \mathcal{H}(b_1) = \mathcal{L}\big(a_1 \vee \mathcal{H}(b_1)\big),$$
$$\mathcal{H}\big(\mathcal{L}\big(a_1 \vee \mathcal{H}(b_1)\big)\big) = \mathcal{H} \circ \mathcal{L} \circ \mathcal{H}(b_1) = \mathcal{H}(b_1) = a_1 \vee \mathcal{H}(b_1).$$

Hence, $(a_1 \vee \mathcal{H}(b_1), \mathcal{L}(a_1 \vee \mathcal{H}(b_1)))$ is a sufficient and necessary information granule.
(2) This item can be obtained similarly. □

Case 4. The method to transform the sufficient information granules into sufficient and necessary information granules can be represented in the following.

Theorem 9. *Let $(L_1, L_2, \mathcal{L}, \mathcal{H})$ be a cognitive system and $\mathcal{G}_2$ be a sufficient information granule set. If $(a_1, b_1) \in \mathcal{G}_2$,*

(1) $(a_1 \wedge \mathcal{H}(b_1), \mathcal{L}(a_1 \wedge \mathcal{H}(b_1))) \in \mathcal{G}_1 \cap \mathcal{G}_2$;
(2) $(\mathcal{H}(b_1 \wedge \mathcal{L}(a_1)), b_1 \wedge \mathcal{L}(a_1)) \in \mathcal{G}_1 \cap \mathcal{G}_2$.

Proof. (1) By $(a_1, b_1) \in \mathcal{G}_2$, we have $\mathcal{L}(a_1) \leqslant b_1$ and $\mathcal{H}(b_1) \leqslant a_1$.
Thus,

$$a_1 \wedge \mathcal{H}(b_1) = \mathcal{H}(b_1), \qquad \mathcal{L}\big(a_1 \wedge \mathcal{H}(b_1)\big) = \mathcal{L} \circ \mathcal{H}(b_1).$$

Thus, from the above discussion and Theorem 1, we have the following:

$$\mathcal{L}\big(a_1 \wedge \mathcal{H}(b_1)\big) = \mathcal{L} \circ \mathcal{H}(b_1),$$
$$\mathcal{H}\big(\mathcal{L}\big(a_1 \wedge \mathcal{H}(b_1)\big)\big) = \mathcal{H} \circ \mathcal{L} \circ \mathcal{H}(b_1) = \mathcal{H}(b_1).$$

Hence, $(a_1 \wedge \mathcal{H}(b_1), \mathcal{L}(a_1 \wedge \mathcal{H}(b_1)))$ is a sufficient and necessary information granule.
(2) This item can be obtained similarly. □

From the above ways, we can come to a conclusion as follows: there are six methods to transform the general information granules into necessary information granules; there are two methods to transform necessary information granules into sufficient and necessary information granules (Fig. 11); there are two methods to transform the general information granules into sufficient information granules; there are two methods to transform sufficient information granules into sufficient and necessary information granules (Fig. 12). So we can obtain 16 methods to transform the general information granules into sufficient and necessary information granules.

Theorem 10. *Let (U, A, I) be a formal context where $U = \{x_1, x_2, x_3, \ldots, x_n\}$, $A = \{a_1, a_2, \ldots, a_m\}$, $X \subseteq U$, and $B \subseteq A$. If $B \subseteq X^*$ and $X \subseteq B^*$, $(X' = X \cup B^*,\ B' = X'^*)$ or $(X' = B'^*,\ B' = B \cup X^*)$ or $(B' = X'^*,\ X' = X \cap B^*)$ or $(X' = B'^*,\ B' = B \cap X^*)$, then (X', B') is a formal concept.*

Proof. It can be proven easily. □

Example 2. (Continued from Example 1.) If we take $a_0 = \{x_1, x_4\}$, $b_0 = \{a, b\}$ from Example 1, then (a_0, b_0) is obviously an inconsistent granule. Thus, we can grasp the necessary information granules $(\{x_2, x_4\}, a)$ and $(\{x_4\}, a)$; the sufficient information granule $(\{x_1, x_2, x_4\}, ab)$. Moreover, we can obtain the sufficient and necessary information granule $(\{x_2, x_4\}, ab)$. Simultaneously, we can first transform the inconsistent granule (a_0, b_0) into a sufficient information granule $(\{x_1, x_2, x_4\}, ab)$, then obtain the sufficient and necessary information granules $(\{x_2, x_4\}, ab)$ and $(\{x_1, x_2, x_4\}, a)$.

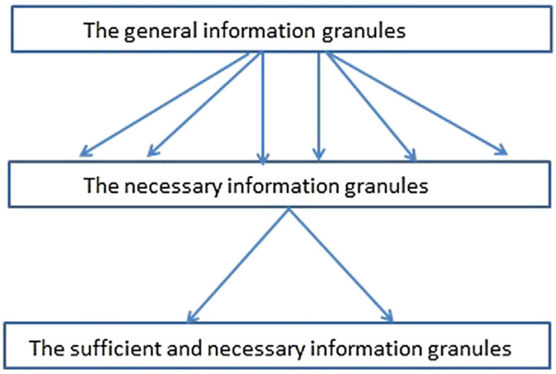

Fig. 11. The transformation of information granules.

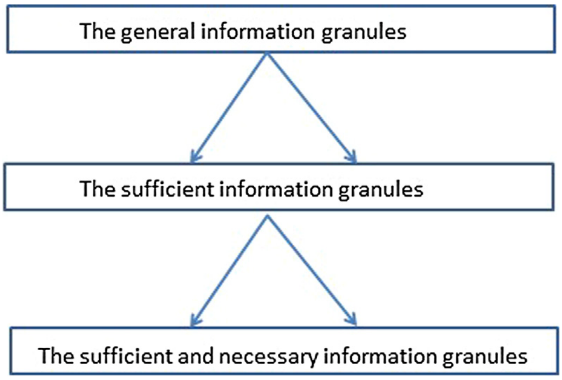

Fig. 12. The transformation of information granules.

6. Algorithm and case study of transformation

According to the above theory proposed, in the next, we can convert an arbitrary information granule into necessary, sufficient, sufficient and necessary information granules by using the following algorithm.

Algorithm. The algorithm of transformation of information granules in cognitive system is described as follows:

Input: An information table and arbitrary information granule in cognitive system.
Output: Necessary information granules, sufficient information granules, sufficient and necessary information granules.

Step 1: Load the data of information table, do initialized setting and compute number of objects and attributes.
Step 2: Input the arbitrary information granule (a_1, b_1) and select randomly one of the two channels.
Step 3: Necessary information granule channel is selected. If the information granule isn't necessary, then skip to step 5. Otherwise, turn to step 7.
Step 4: Sufficient information granule channel is selected. If the information granule isn't sufficient, then skip to step 6. Otherwise, go to step 8.
Step 5: Transform the general information granule (a_1, b_1) into necessary information granules $(a_2^1, b_2^1), (a_2^2, b_2^2), \dots, (a_2^m, b_2^m)$ by choosing one of six methods in Case 1.
Step 6: Transform the general information granule into sufficient information granules $(a_3^1, b_3^1), (a_3^2, b_3^2), \dots, (a_3^n, b_3^n)$ by choosing one of two methods in Case 2.
Step 7: Transform the necessary information granules into sufficient and necessary information granules $(a_4^1, b_4^1), (a_4^2, b_4^2), \dots, (a_4^r, b_4^r)$ by choosing the methods in Case 3.
Step 8: Transform the sufficient information granules into sufficient and necessary information granules $(a_5^1, b_5^1), (a_5^2, b_5^2), \dots, (a_5^s, b_5^s)$ by choosing the methods in Case 4.
Step 9: Output necessary information granules, sufficient information granules, sufficient and necessary information granules. And finish the algorithm.

Experimental computing program can be designed and carried out so as to apply the algorithm studied more directly and practically in this paper. Let N_i $(i = 1, 2, 3, 4, 5, 6)$, S_i $(i = 1, 2)$, C_i $(i = 1, 2)$, D_i $(i = 1, 2)$ stand for the methods in Cases 1, 2, 3, 4, respectively. The main process of the program will be introduced by the flow chart (Fig. 13) in this section and cases are employed to verify the program.

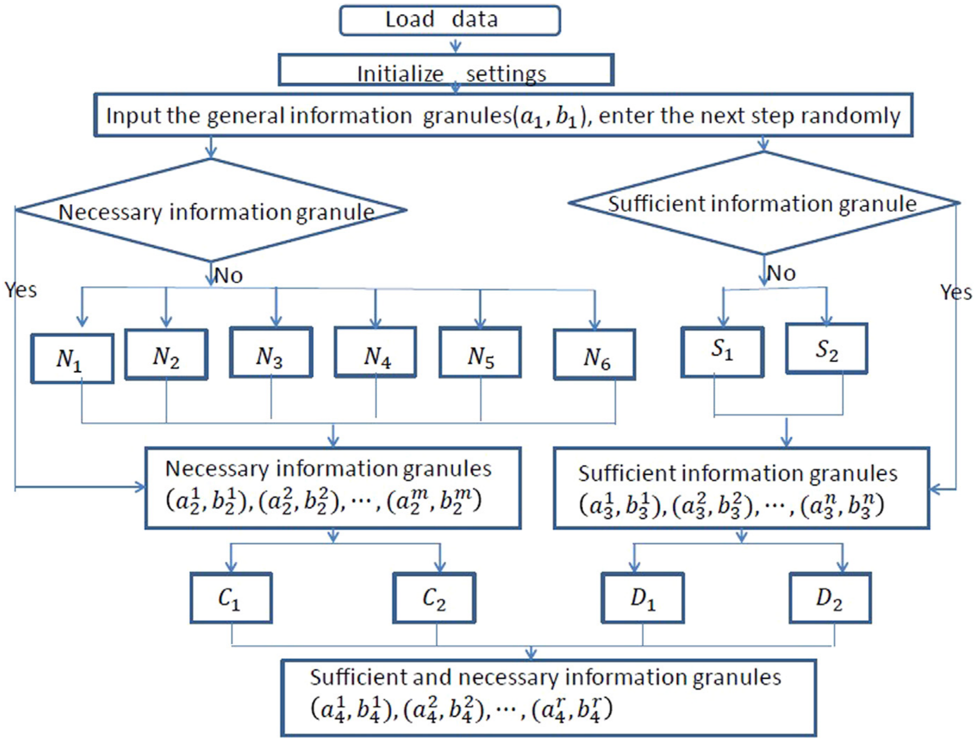

Fig. 13. The program flow chart of the transformation of information granules.

This experimental computing program is running on a personal computer with the following hardware and software.

Name	Model	Parameters
CPU	Intel i5-2410	2.3 GHz
Memory	Samsung DDR3 SDRAM	2 × 2 GB, 1333 MHz
Hard disk	West Data	640 GB
System	Windows 7	32 bit
Platform	C++	6.0

Example 3. A formal context about situations of developing countries is presented in Table 2. The formal context is denoted by (U, A, I), where A is an attribute set. There are 110 objects which represent the kinds of developing countries. The data are from Ref. [29]. The interpretations of the attributes will be listed as follows:

a_1 – Group of 77
a_2 – Non-aligned
a_3 – LLDC (Least Less Developed Country)
a_4 – MSAC (Most Seriously Affected Country)
a_5 – OPEC (Organization of Petroleum Exporting Countries)
a_6 – ACP (African, Caribbean, and Pacific Associables)

If the United Nations plan to grant loans for developing countries in order to help them with economic development. Now, there are 128 developing countries in Table 2. The United Nations need fully consider political and economic environment for the distribution of equity. That's to say, the United Nations must take attribute set A into account when they make decisions which countries to choose. The method proposed in this paper can be used to select the candidate countries.

Suppose (X_0, B_0) is the initial information granule, where X_0 and B_0 are the countries and attributes voted by delegation. But the result (X_0, B_0) may induce the condition that the countries selected don't satisfy the given attributes and the countries satisfying the given attributes are not selected. Now let $X_0 = \{x_1, x_{11}, x_{14}, x_{35}, x_{47}, x_{52}, x_{59}, x_{78}, x_{84}, x_{92}, x_{87}, x_{95}, x_{106}\}$, $B_0 = \{a_1, a_2, a_3, a_6\}$.

If the funding is limited, the United Nations only select the countries which must satisfy the given condition attribute. Now the necessary information granule is a good choice. The information granule $(X_0, B_0) \in \mathcal{G}_1$ is necessary information because of $B_0 \leqslant \mathcal{L}(X_0)$ through the above methods. That's to say, these countries X_0 satisfy and precede the given condition attributes B_0.

Table 2
(U, A, I) about situations of developing countries.

	Countries	Group of 77	Non-aligned	LLDC	MSAC	OPEC	ACP
x_1	Afghanistan	1	1	1	1	0	0
x_2	Algeria	1	1	0	0	1	0
x_3	Angola	1	1	0	0	0	1
x_4	Antigua and Barbuda	1	0	0	0	0	1
x_5	Argentina	1	0	0	0	0	0
x_6	Bahamas	1	0	0	0	0	1
x_7	Bahrain	1	1	0	0	0	0
x_8	Bangladesh	1	1	1	1	0	0
x_9	Barbados	1	1	0	0	0	1
x_{10}	Belize	1	1	0	0	0	1
x_{11}	Benin	1	1	1	1	0	1
x_{12}	Bhutan	1	1	1	0	0	0
x_{13}	Bolivia	1	1	0	0	0	0
x_{14}	Botswana	1	1	1	0	0	1
x_{15}	Brazil	1	0	0	0	0	0
x_{16}	Brunei	0	0	0	0	0	0
x_{17}	Burkina Faso	1	1	1	1	0	1
x_{18}	Burundi	1	1	1	1	0	1
x_{19}	Cambodia	1	1	0	1	0	0
x_{20}	Cameroon	1	1	0	1	0	1
x_{21}	Cape Verde	1	1	1	1	0	1
x_{22}	Central African Rep.	1	1	1	1	0	1
x_{23}	Chad	1	1	1	1	0	1
x_{24}	Chile	1	0	0	0	0	0
x_{25}	China	0	0	0	0	0	0
x_{26}	Colombia	1	1	0	0	0	0
x_{27}	Comoros	1	1	1	0	0	1
x_{28}	Congo	1	1	0	0	0	1
x_{29}	Costa Rica	1	0	0	0	0	0
x_{30}	Cuba	1	1	0	0	0	0
x_{31}	Djibouti	1	1	1	0	0	1
x_{32}	Dominica	1	1	0	0	0	1
x_{33}	Dominican Rep.	1	0	0	0	0	1
x_{34}	Ecuador	1	1	0	0	1	0
x_{35}	Egypt	1	1	0	1	0	0
x_{36}	El Salvador	1	0	0	1	0	0
x_{37}	Equatorial Guinea	1	1	1	0	0	1
x_{38}	Ethiopia	1	1	1	1	0	1
x_{39}	Fiji	1	0	0	0	0	1
x_{40}	Gabon	1	1	0	0	1	1
x_{41}	Gambia	1	1	1	1	0	1
x_{42}	Ghana	1	1	1	1	0	1
x_{43}	Grenada	1	1	0	0	0	1
x_{44}	Guatemala	1	0	0	1	0	0
x_{45}	Guinea	1	1	1	1	0	1
x_{46}	Guinea-Bissau	1	1	1	1	0	1
x_{47}	Guyana	1	1	0	1	0	1
x_{48}	Haiti	1	0	1	1	0	1
x_{49}	Honduras	1	0	0	1	0	0
x_{50}	India	1	1	0	1	0	0
x_{51}	Indonesia	1	1	0	0	1	0
x_{52}	Iran	1	1	0	0	1	0
x_{53}	Iraq	1	1	0	0	1	0
x_{54}	Ivory Coast	0	1	0	1	0	1
x_{55}	Jamaica	1	1	0	0	0	1
x_{56}	Jordan	l	1	0	0	0	0
x_{57}	Kenya	1	1	0	1	0	1
x_{58}	Kiribati	0	0	1	0	0	1
x_{59}	Korea-North	1	1	1	0	0	0
x_{60}	Korea-South	1	0	0	0	0	0
x_{61}	Kuwait	1	1	0	0	1	0
x_{62}	Laos	1	1	1	1	0	0
x_{63}	Lebanon	1	1	0	0	0	0
x_{64}	Lesotho	1	1	1	1	0	1
x_{65}	Liberia	1	1	0	0	0	1
x_{66}	Libya	1	1	0	0	1	0
x_{67}	Madagascar	1	1	1	1	0	1
x_{68}	Malawi	1	1	1	0	0	1
x_{69}	Malaysia	1	1	0	0	0	0

(*continued on next page*)

Table 2 (*Continued*)

	Countries	Group of 77	Non-aligned	LLDC	MSAC	OPEC	ACP
x_{70}	Maldives	1	1	1	0	0	0
x_{71}	Mali	1	1	1	1	0	1
x_{72}	Mauritania	1	1	1	1	0	1
x_{73}	Mauritius	1	1	0	0	0	1
x_{74}	Mexico	1	0	0	0	0	0
x_{75}	Mongolia	0	0	1	0	0	0
x_{76}	Morocco	1	1	0	0	0	0
x_{77}	Mozambique	1	1	0	1	0	1
x_{78}	Myanmar	1	0	1	1	0	0
x_{79}	Namibia	1	0	0	0	0	1
x_{80}	Nauru	0	0	0	0	0	0
x_{81}	Nepal	1	1	1	1	0	0
x_{82}	Nicaragua	1	1	0	0	0	0
x_{83}	Niger	1	1	1	1	0	1
x_{84}	Nigeria	1	1	0	0	1	1
x_{85}	Oman	1	1	0	0	0	0
x_{86}	Pakistan	1	1	0	1	0	0
x_{87}	Panama	1	1	0	0	0	0
x_{88}	Papua New Guinea	1	0	0	0	0	1
x_{89}	Paraguay	1	0	0	0	0	0
x_{90}	Peru	1	1	0	0	0	0
x_{91}	Philippines	1	0	0	0	0	0
x_{92}	Qatar	1	1	0	0	1	0
x_{93}	Reunion	0	0	0	0	0	0
x_{94}	Rwanda	1	1	1	1	0	1
x_{95}	Samoa	1	0	1	1	0	1
x_{96}	Sao Tome e Principe	1	1	1	0	0	1
x_{97}	Saudi Arabia	1	1	0	0	1	0
x_{98}	Senegal	1	1	0	1	0	1
x_{99}	Seychelles	1	1	0	0	0	1
x_{100}	Sierra Leone	1	1	1	1	0	1
x_{101}	Singapore	1	1	0	0	0	0
x_{102}	Solomon Islands	1	0	0	0	0	1
x_{103}	Somalia	1	1	1	1	0	1
x_{104}	Sri Lanka	1	1	0	1	0	0
x_{105}	St Kitts	0	0	0	0	0	0
x_{106}	St Lucia	1	1	0	0	0	1
x_{107}	St Vincent Grenada	1	0	0	0	0	1
x_{108}	Sudan	1	1	1	1	0	1
x_{109}	Surinam	1	1	0	0	0	1
x_{110}	Swaziland	1	1	0	0	0	1
x_{111}	Syria	1	1	0	0	0	0
x_{112}	Tanzania	1	1	1	1	0	1
x_{113}	Thailand	1	0	0	0	0	0
x_{114}	Togo	1	1	1	0	0	1
x_{115}	Tonga	1	0	0	0	0	1
x_{116}	Trinidad and Tobago	1	1	0	0	0	1
x_{117}	Tunisia	1	1	0	0	0	0
x_{118}	Tuvalu	0	0	1	0	0	1
x_{119}	Uganda	1	1	1	1	0	1
x_{120}	United Arab Emirates	1	1	0	0	1	0
x_{121}	Uruguay	1	0	0	0	0	0
x_{122}	Vanuatu	1	1	1	0	0	1
x_{123}	Venezuela	1	1	0	0	1	0
x_{124}	Vietnam	1	1	1	0	0	0
x_{125}	Yemen	1	1	1	1	0	0
x_{126}	Zaire	1	1	1	0	0	1
x_{127}	Zambia	1	1	1	0	0	1
x_{128}	Zimbabwe	1	1	0	0	0	1

If the United Nations try to consider as many countries as possible, they may relax the conditions to some developing countries. Now the sufficient information granule is a good choice. Let (X_0, B_0) be the initial information granule, then we can compute the sufficient information granules of (X_0, B_0) by the above program as follows:

$$(X_1, B_1) \in \mathcal{G}_2, \qquad (X_2, B_2) \in \mathcal{G}_2,$$

where

$$X_1 = \{x_1, x_{11}, x_{14}, x_{17}, x_{18}, x_{21}, x_{22}, x_{23}, x_{27}, x_{31}, x_{35}, x_{37}, x_{38}, x_{41}, x_{42}, x_{45}, x_{46}, x_{47}, x_{52}, x_{59}, x_{64},$$
$$x_{67}, x_{68}, x_{71}, x_{72}, x_{78}, x_{83}, x_{84}, x_{92}, x_{94}, x_{95}, x_{96}, x_{100}, x_{103}, x_{106}, x_{108}, x_{112}, x_{114}, x_{119}, x_{122}, x_{126}, x_{127}\},$$

$$B_1 = \{a_1, a_2, a_3, a_6\},$$

$$X_2 = U - \{x_{16}, x_{25}, x_{58}, x_{75}, x_{80}, x_{93}, x_{105}, x_{118}\},$$

$$B_2 = \{a_1, a_2, a_3, a_6\}.$$

From the sufficient information granules (X_1, B_1) and (X_2, B_2), we can know that the countries X_1, X_2 only satisfy or don't satisfy the attributes B_1 respectively.

If the United Nations hope that the selected countries must satisfy the given attributes and all the countries which satisfy the given attributes must be selected. Now the sufficient and necessary information granule is a good choice. Thus, we can compute the sufficient and necessary information granules of (X_0, B_0) by the above program as follows:

$$(X_3, B_3) \in \mathcal{G}_1 \cap \mathcal{G}_2, \qquad (X_4, B_4) \in \mathcal{G}_1 \cap \mathcal{G}_2,$$

where

$$X_3 = \{x_{11}, x_{14}, x_{17}, x_{18}, x_{21}, x_{22}, x_{23}, x_{27}, x_{31}, x_{37}, x_{38}, x_{41}, x_{42}, x_{45}, x_{46}, x_{64}, x_{67},$$
$$x_{68}, x_{71}, x_{72}, x_{83}, x_{94}, x_{96}, x_{100}, x_{103}, x_{108}, x_{112}, x_{114}, x_{119}, x_{122}, x_{126}, x_{127}\},$$

$$B_3 = \{a_1, a_2, a_3, a_6\}.$$

$$X_4 = U - \{x_{16}, x_{25}, x_{58}, x_{75}, x_{80}, x_{93}, x_{105}, x_{118}\}$$

$$B_4 = \{a_1\}.$$

From the sufficient and necessary information granules (X_3, B_3) and (X_4, B_4), we can know that the countries X_3, X_4 satisfy the given attributes B_3, B_4 and all the countries which satisfy the B_3, B_4 be selected respectively.

7. Conclusions

A key event in human cognitive processes is understanding the necessary and sufficient attributes of an object when getting to know it. Once the sufficient and necessary attributes of the object are obtained, the object can be completely grasped. However, the cognitive process is very complex. To describe the cognitive process, this paper constructed a novel model of cognitive systems based on formal concept analysis. We proposed two operators between an object and its attributes. Necessary information, sufficient information, and sufficient and necessary information granules were also presented. Moreover, we showed an approach to transform arbitrary information granules into necessary information granules, sufficient information granules, sufficient and necessary information granules. Finally, experiments were implemented to illustrate the algorithm designed in this paper. In particular, the capability of the proposed method will be very useful in the analysis of cognitive systems with big data.

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